4.14 Logarithmic and Exponential Equations

Solving Logarithmic Equations

- To avoid extraneous solutions, determine the domain of the variable first.
- Use the properties of logarithms to manipulate the equations.
- Try rewriting as an exponential function: $y = \log_a x \Leftrightarrow x = a^y$
- Remember the property: $\log_a M = \log_a N \Leftrightarrow M = N$

Examples:

a) $\log_3(3x-1) = 2$

b) $-2\log_4 x = \log_4 9$

c) $3\log_2(x-1) + \log_2 4 = 5$

d) $\ln(x+1) - \ln(x) = 2$

e) $\log_6(x+4) + \log_6(x+3) = 1$

f) $\log_a x + \log_a(x-2) = \log_a(x+4)$

Solving Exponential Equations

- If possible, make the bases the same, set exponents equal, and solve: $a^u = a^v \Leftrightarrow u = v$.
- If bases cannot be made the same, use algebraic techniques or rewrite as a log and use log properties.
- Remember the property: $M = N \Leftrightarrow \log_a M = \log_a N$
- If an exact solution cannot be found, use a graphing utility to obtain an approximate solution.

Examples:

a) $2^{-x} = 1.5$

b) $0.3\left(4^{0.2x}\right) = 0.2$

c) $e^{x+3} = \pi^{x}$

d) $6^{x-4} = 11^{5x+1}$

e) $36^{x} - 6(6)^{x} = -9$

f) $2(49)^{x} + 11(7)^{x} + 5 = 0$

Solving by Graphing

1. Let each side of the equal sign be a separate function. Set one as Y_1 and the other as Y_2.
2. Graph Y_1 and Y_2 on your graphing calculator.
3. Find the intersection of the two graphs.
4. To solve an inequality, determine the interval of *x*-values that make the statement true.

a) $\dfrac{e^{x} - e^{-x}}{2} = 2$

b) $5^{x-1} > 2^{x+1}$